3 $f(x,y) = \frac{3}{8}(1+x)$, $-1 < y < x < 1$ $\qquad S = \frac{E(XZ) - EX\,EZ}{\sqrt{DX}\,\sqrt{DZ}}$

I наин: $f_x(x) = \int_{-1}^{x} \frac{3}{8}(1+x)\,dy = \frac{3}{8}(1+x)^2$, $-1 < x < 1$

$EX = \int_{-1}^{1} \frac{3}{8} x(1+x)^2 dx = \frac{3}{8}\int_{-1}^{1}(x + 2x^2 + x^3)\,dx$

$= \frac{3}{8} 2\frac{2}{3} = \boxed{\frac{1}{2}}$

$EX^2 = \int_{-1}^{1} \frac{3}{8} x^2(1+x)^2 dx = \frac{3}{8}\int_{-1}^{1}(x^2 + 2x^3 + x^4)\,dx$

$= \frac{3}{8}\left(\frac{2}{3} + \frac{2}{5}\right) = \frac{3}{8}\cdot\frac{2\cdot 8}{15} = \boxed{\frac{2}{5}}$

$DX = \frac{2}{5} - \frac{1}{4} = \boxed{\frac{3}{20}}$

$EZ = E(X+Y) = EX + EY = \frac{1}{2} + EY$ $\qquad y \in (-1,1)$

$f_y(y) = \int_{y}^{1} \frac{3}{8}(1+x)\,dx = \frac{3}{8}\left(1 - y + \frac{1}{2} - \frac{y^2}{2}\right) = \frac{3}{16}(3 - 2y - y^2)$

$EY = \frac{3}{16}\int_{-1}^{1}(3 - 2y - y^2)\,y\,dy = \frac{3}{16}\left(-2\frac{2}{3}\right) = -\frac{1}{4}$

$EZ = \frac{1}{2} - \frac{1}{4} = \boxed{\frac{1}{4}}$

$E(XZ) = E(X(X+Y)) = EX^2 + E(XY)$

$E(XY) = \int_{-1}^{1}\int_{-1}^{x} xy\,\frac{3}{8}(1+x)\,dy\,dx = \frac{3}{8}\int_{-1}^{1}(x + x^2)\left(\frac{x^2}{2} - \frac{1}{2}\right)dx$

$= \frac{3}{16}\int_{-1}^{1}(x^3 - x + x^4 - x^2)\,dx = \frac{3}{16}\left(\frac{2}{5} - \frac{2}{3}\right) = -\frac{3}{16}\cdot\frac{4}{15} = \boxed{-\frac{1}{20}}$

$E(XZ) = \frac{2}{5} - \frac{1}{20} = \frac{8-1}{20} = \boxed{\frac{7}{20}}$

$\operatorname{cov}(X,Z) = \frac{7}{20} - \frac{1}{2}\cdot\frac{1}{4} = \frac{14-5}{40} = \frac{9}{40}$

$DZ = DX + DY + 2\operatorname{cov}(X,Y)$

$EY^2 = \frac{3}{16}\int_{-1}^{1} y^2(3 - 2y - y^2)\,dy = \frac{3}{16}\left(2 - \frac{2}{5}\right) = \frac{3}{16}\cdot\frac{8}{5} = \frac{3}{10}$

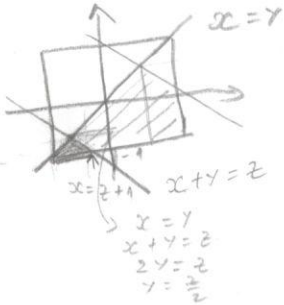
$DY = \frac{3}{10} - \frac{1}{16} = \frac{24-5}{80} = \boxed{\frac{19}{80}}$

$\operatorname{cov}(X,Y) = E(XY) - EX\,EY = -\frac{1}{20} + \frac{1}{2}\cdot\frac{1}{4} = \frac{5-2}{40} = \frac{3}{40}$

$DZ = \frac{3}{20} + \frac{19}{80} + \frac{3}{20} = \frac{12+19+12}{80} = \frac{43}{80}$

$S_{x,z} = \frac{\frac{7}{20} - \frac{1}{2}\cdot\frac{1}{4}}{\sqrt{\frac{3}{20}}\sqrt{\frac{43}{80}}} = \frac{19}{\sqrt{129}} = 0.79$

II наин:



$x = y$
$x + y = z$
$2y = z$
$y = \frac{z}{2}$

$F_z(z) = P\{Z \le z\} = P\{X + Y \le z\} = (*)$

1° $z \in [-2, 0]$: $(*) = \int_{-1}^{\frac{z}{2}}\int_{y}^{z-y} \frac{3}{8}(1+x)\,dx\,dy$

$= \frac{3}{8}\int_{-1}^{\frac{z}{2}}\left(z - y - y + \frac{(z-y)^2 - y^2}{2}\right)dy$

$= \frac{3}{8}\int_{-1}^{\frac{z}{2}}\left(z - 2y + \frac{z^2}{2} - zy\right)dy$

$= \frac{3}{8}\left(z\left(\frac{z}{2} - 1\right) - \frac{z^2}{4} + 1 + \frac{z^2}{2}\left(\frac{z}{2} - 1\right) - z\frac{z^2}{8} + \frac{z}{2}\right)$

$= \frac{3}{8}\left(\frac{z^2}{2} - z - \frac{z^2}{4} + 1 + \frac{z^3}{4} - \frac{z^2}{2} - \frac{z^3}{8} + \frac{z}{2}\right)$

$= \frac{3}{64}z^3 - \frac{3}{32}z^2 + \frac{3}{16}z + \frac{3}{8}$

$= \frac{3}{64}(z^3 + 6z^2 + 12z + 8) = \frac{3}{64}(z+2)^3$

2° $z \in (0, 2]$: $(*) = 1 - \int_{\frac{z}{2}}^{1}\int_{z-x}^{x}\frac{3}{8}(1+x)\,dy\,dx = 1 - \frac{3}{8}\int_{\frac{z}{2}}^{1}(1+x)(2x - z)\,dx$

$2x - z + 2x^2 - zx$

$= 1 - \frac{3}{8}\left(1 - \frac{z^2}{4} - z + \frac{z^2}{2} + \frac{2}{3} - \frac{z}{3}\frac{z^3}{8} - \frac{z}{2} + \frac{z}{2}\frac{z^2}{4}\right)$