$$f_Z(z) = \begin{cases} \frac{3}{64}3(z+2)^2, & z \in [-2,0] \\ -\frac{3z^2}{64} - \frac{6}{32}z + \frac{9}{16}, & z \in (0,2] \\ 0, & \text{иначе} \end{cases}$$

$$EZ = \int_{-2}^{0} \frac{9}{64}(z^2+4z+4)z\,dz + \int_0^2 \left(-\frac{3}{64}z^2 - \frac{3}{16}z + \frac{9}{16}\right)z\,dz$$

$$= \frac{9}{64}\left(-\frac{2^4}{4} + 4\frac{2^3}{3} - 4\frac{4}{2}\right) - \frac{3}{64}\frac{2^4}{4} - \frac{3}{16}\frac{2^3}{3} + \frac{9}{16}\frac{4}{2}$$

$$= -\frac{9}{16} + \frac{3}{2} - \cancel{\frac{9}{8}} - \frac{3}{16} - \frac{1}{2} + \cancel{\frac{9}{8}} = -\frac{12}{16} + 1 = \boxed{\frac{1}{4}}$$

$$EZ^2 = \int_{-2}^{0} \frac{9}{64}(z^2+4z+4)z^2\,dz + \int_0^2 \left(-\frac{3}{64}z^2 - \frac{3}{16}z + \frac{9}{16}\right)z^2\,dz$$

$$= \frac{9}{64}\left(\frac{2^5}{5} - 4\frac{2^4}{4} + 4\frac{2^3}{3}\right) - \frac{3}{64}\frac{2^5}{5} - \frac{3}{16}\frac{2^4}{4} + \frac{9}{16}\frac{2^3}{3}$$

$$= \frac{9}{10} - \frac{9}{4} + \frac{3}{2} - \frac{3}{10} - \frac{3}{4} + \frac{3}{2} = \frac{6}{10} - \cancel{3} + \cancel{3} = \frac{3}{5}$$

$$DZ = EZ^2 - (EZ)^2 = \frac{3}{5} - \frac{1}{16} = \frac{48-5}{80} = \boxed{\frac{43}{80}}$$

1 (5) $P = 0.7 \cdot 0.9 + 0.3 \cdot \frac{1}{3} = 0.63 + 0.1 = 0.73$ вероватноћа да тачно одговори

A - зна одговор

$P = P(A) + P(\bar{A}) \cdot \frac{1}{3}$

X - студент тачно одговара

A - тачно реши задатак

H_1 - мисли да зна

H_2 - мисли да не зна

$P(A) = P(H_1)P(A|H_1) + P(H_2)P(A|H_2)$

$= 0.7 \cdot 0.9 + 0.3 \cdot \frac{1}{3} = 0.63 + 0.1$

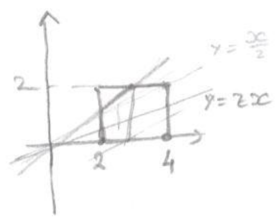
$= 0.73$

(5) $X \in \mathcal{B}(20, p)$

$$P\{X = 15\} = \binom{20}{15} 0.73^{15} 0.27^5 = \binom{20}{5} 0.73^{15} 0.27^5 = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16}{5 \cdot 4 \cdot 3 \cdot 2} 0.73^{15} 0.27^5$$

≈ 0.20

2 $f(x,y) = \begin{cases} \frac{1}{4}y, & (x,y) \in D \\ 0, & \text{иначе} \end{cases}$



$y = 2$

$y = zx$

$x = \frac{2}{z}$

$$F_Z(z) = P\{Z \le z\} = P\left\{\frac{Y}{X} \le z\right\} = P\{Y \le zX\} = (*)$$

1° $z \in [0, \frac{1}{2}]$: $(*) = \int_2^4 \int_0^{zx} \frac{y}{4}\,dy\,dx$

$$= \frac{1}{4}\int_2^4 \frac{z^2x^2}{2}\,dx = \frac{1}{8}z^2 \frac{x^3}{3}\Big|_2^4 = \frac{56}{3 \cdot 8}z^2 = \frac{7}{3}z^2$$

2° $z \in (\frac{1}{2}, 1]$: $(*) = 1 - \int_2^{\frac{2}{z}} \int_{zx}^{2} \frac{1}{4}y\,dy\,dx$

$$= 1 - \frac{1}{4}\int_2^{\frac{2}{z}} \frac{4 - z^2x^2}{2}\,dx$$

$$= 1 - \frac{1}{2}\left(\frac{2}{z} - 2\right) + \frac{1}{8}z^2\frac{x^3}{3}\Big|_2^{\frac{2}{z}}$$

$$= 1 - \frac{1}{z} + 1 + \frac{1}{3 \cdot 8}z^2\frac{8}{z^3} - \frac{1}{3 \cdot 8}z^2 \cdot 8$$

$$= 2 - \frac{2}{3z} - \frac{z^2}{3} = -\frac{z^3 + 6z + 2}{3z}$$

$$F(z) = \begin{cases} 0, & z < 0 \\ \frac{7}{3}z^2, & z \in [0, \frac{1}{2}) \\ -\frac{z^3 - 6z + 2}{3z}, & z \in [\frac{1}{2}, 1] \\ 1, & z > 1 \end{cases}$$

$$2^\circ\ (*) = \int_2^{2/z}\int_0^{zx} \frac{y}{4}\,dy\,dx + \int_{2/z}^4 \int_0^2 \frac{y}{4}\,dy\,dx$$

$$= \frac{1}{4}\int_2^{\frac{2}{z}} \frac{z^2x^2}{2}\,dx + \frac{1}{4}\int_{\frac{2}{z}}^4 \frac{4}{2}\,dx$$

$$= \frac{1}{8}z^2\frac{x^3}{3}\Big|_2^{\frac{2}{z}} + \frac{1}{2}\left(4 - \frac{2}{z}\right)$$

$$= \frac{1}{3 \cdot 8}z^2\frac{8}{z^3} - \frac{1}{8 \cdot 3}z^2 \cdot 8 + 2 - \frac{1}{z}$$