(36)

а) $T_n = \sqrt{n}(2\bar{X} - 1)$ $\quad X \in U[0,\theta]$

$W = \{|T_n| > k\}$ $\quad \bar{X}_n \sim N\left(\frac{\theta}{2}, \frac{\theta^2}{12n}\right)$ $\quad \frac{\bar{X}_n - \frac{\theta}{2}}{\frac{\theta}{2\sqrt{3n}}} = \frac{2\bar{X}_n - \theta}{\frac{\theta}{\sqrt{3n}}} =$

$= \sqrt{3n}\left(2\frac{\bar{X}_n}{\theta} - 1\right) \sim N(0,1)$

$\alpha = P_{H_0}\{|T_n| > k\} = P_{H_0}\{|\sqrt{3n}(2\bar{X} - 1)| > \sqrt{3}k\}$

$\sqrt{3}k = \Phi^{-1}(1 - \frac{\alpha}{2}) = \Phi^{-1}(0.975)$

$k = \frac{1}{\sqrt{3}} 1.96$

б) $M(\theta) = P_\theta\{|T_n| > \frac{1.96}{\sqrt{3}}\} = 1 - P_\theta\{-\frac{1.96}{\sqrt{3}} \le \sqrt{n}(2\bar{X} - 1) \le \frac{1.96}{\sqrt{3}}\}$

$= 1 - P_\theta\{-\frac{1.96}{\sqrt{3n}} + 1 \le 2\bar{X} \le \frac{1.96}{\sqrt{3n}} + 1\}$

$= 1 - P_\theta\{(-\frac{1.96}{\sqrt{3n}} + 1)\frac{1}{\theta} \le \frac{2\bar{X}}{\theta} \le \frac{1}{\theta}(\frac{1.96}{\sqrt{3n}} + 1)\}$

$= 1 - P_\theta\{\frac{1}{\theta}(-\frac{1.96}{\sqrt{3n}} + 1) - 1 \le \frac{2\bar{X}}{\theta} - 1 \le \frac{1}{\theta}(\frac{1.96}{\sqrt{3n}} + 1) - 1\}$

$= 1 - P_\theta\{\sqrt{3n}(\frac{1}{\theta}(-\frac{1.96}{\sqrt{3n}} + 1) - 1) \le X^* \le \sqrt{3n}(\frac{1}{\theta}(\frac{1.96}{\sqrt{3n}} + 1) - 1)\}$

$= 1 - \int_{-\frac{1.96}{\theta} + \frac{\sqrt{3n}}{\theta} - \sqrt{3n}}^{\frac{1.96}{\theta} + \frac{\sqrt{3n}}{\theta} - \sqrt{3n}} \varphi(x)\,dx$

$M_\theta'(\theta) = +\varphi(\frac{1.96}{\theta} + \frac{\sqrt{3n}}{\theta} - \sqrt{3n})\frac{1.96 + \sqrt{3n}}{\theta^2} - \varphi(-\frac{1.96}{\theta} + \frac{\sqrt{3n}}{\theta} - \sqrt{3n})\frac{-1.96 + \sqrt{3n}}{\theta^2}$

$= 0$

$\frac{1.96 + \sqrt{3n}}{\theta^2}\frac{1}{\sqrt{2\pi}} e^{-(\frac{1.96}{\theta} + \frac{\sqrt{3n}}{\theta} - \sqrt{3n})^2 \cdot \frac{1}{2}} - \frac{-1.96 + \sqrt{3n}}{\theta^2}\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{-1.96}{\theta} + \frac{\sqrt{3n}}{\theta} - \sqrt{3n})} > 0$

$e^{-\frac{1}{2}(\frac{19.28}{\theta^2} - 17.32)^2} > e^{-\frac{1}{2}(\frac{15.36}{\theta} - 17.32)^2}$

$(\frac{19.28}{\theta} - 17.32)^2 < (\frac{15.36}{\theta} - 17.32)^2$

$(\frac{19.28}{\theta} - 17.32 - \frac{15.36}{\theta} + 17.32)\cdot(\frac{19.28}{\theta} - 17.32 + \frac{15.36}{\theta} - 17.32) < 0$

$\frac{3.92}{\theta}(\frac{34.64}{\theta} - 34.64) = 0 \Rightarrow \theta = 1$

За $\theta > 1$: $\varphi(\frac{19.28}{\theta^2} - 17.32) - \varphi(\frac{15.36}{\theta} - 17.32) > 0$

$\Rightarrow M_\theta'(\theta) > 0, \forall \theta > 1$

$\Rightarrow M(\theta)$ расућа функција

$\Rightarrow \min M(\theta) = M(1) = 0.05$

$\Rightarrow$ тест је непристрасан