$\boxed{84}$

$$I(\theta) = -E\left(\frac{\partial^2 \ln f(X;\theta)}{\partial\theta^2}\right) = E\left(\frac{\partial^2}{\partial\theta^2}\left(\ln\pi + \ln(1+(x-\theta)^2)\right)\right)$$

$$= E\left(\frac{\partial}{\partial\theta}\left(\frac{-2(x-\theta)}{1+(x-\theta)^2}\right)\right) = 2\,E\left(\frac{1+(x-\theta)^2+(\theta-x)\cdot 2(x-\theta)}{(1+(x-\theta)^2)^2}\right)$$

$$= 2\,E\left(\frac{1}{1+(x-\theta)^2} - \frac{2(x-\theta)^2}{(1+(x-\theta)^2)^2}\right)$$

$$= 2\int_{-\infty}^{+\infty}\frac{1}{1+(x-\theta)^2}\cdot\frac{1}{\pi}\frac{1}{1+(x-\theta)^2}\,dx - 4\int_{-\infty}^{+\infty}\frac{(x-\theta)^2}{(1+(x-\theta)^2)^2}\,\frac{1}{\pi}\,\frac{1}{1+(x-\theta)^2}\,dx$$

$$= \frac{2}{\pi}\int_{-\infty}^{+\infty}\frac{1}{(1+(x-\theta)^2)^2}\,dx - \frac{4}{\pi}\int_{-\infty}^{+\infty}\frac{(x-\theta)^2}{(1+(x-\theta)^2)^3}\,dx = \left|\begin{matrix} x-\theta = t \\ dx = dt\end{matrix}\right|$$

$$= \frac{2}{\pi}\int_{-\infty}^{+\infty}\frac{1}{(1+t^2)^2}\,dt - \frac{4}{\pi}\int_{-\infty}^{+\infty}\frac{t^{2+1-1}}{(1+t^2)^3}\,dt$$

$$= \frac{2}{\pi}\int_{-\infty}^{+\infty}\frac{1}{(1+t^2)^2}\,dt - \frac{4}{\pi}\int_{-\infty}^{+\infty}\frac{1}{(1+t^2)^2}\,dt + \frac{4}{\pi}\int_{-\infty}^{+\infty}\frac{1}{(1+t^2)^3}\,dt$$

$$= -\frac{2}{\pi}\int_{-\infty}^{+\infty}\frac{1}{(1+t^2)^2}\,dt + \frac{4}{\pi}\int_{-\infty}^{+\infty}\frac{1}{(1+t^2)^3}\,dt = \frac{4}{\pi}I_3 - \frac{2}{\pi}I_2$$

$$I_k = \int_{-\infty}^{+\infty}\frac{1}{(1+t^2)^k}\,dt = \int_{-\infty}^{+\infty}\frac{1+t^2}{(1+t^2)^{k+1}}\,dt = I_{k+1} + \int_{-\infty}^{+\infty}\frac{t^2}{(1+t^2)^{k+1}}\,dt$$

$$= \left|\begin{matrix} u = t \quad du = dt \\ dv = \dfrac{t\,dt}{(1+t^2)^{k+1}} \\ v = \displaystyle\int\frac{t\,dt}{(1+t^2)^{k+1}} = \left|\begin{matrix}1+t^2 = x \\ 2\cdot t\,dt = dx\end{matrix}\right| \\ = \dfrac{1}{2}\displaystyle\int\frac{dx}{x^{k+1}} = -\frac{1}{2k\,x^k} \\ = -\dfrac{1}{2k(1+t^2)^k}\end{matrix}\right| = I_{k+1} - \cancel{\frac{t}{2k(1+t^2)^k}\Big|_{-\infty}^{+\infty}}^{\,0} + \frac{1}{2k}\int_{-\infty}^{+\infty}\frac{dt}{(1+t^2)^k}$$

$$= I_{k+1} + \frac{1}{2k}I_k$$

$$\Rightarrow\quad I_{k+1} = \left(1-\frac{1}{2k}\right)I_k$$

$$I_1 = \int_{-\infty}^{+\infty}\frac{1}{1+x^2}\,dx = \pi$$

$$I_2 = \left(1-\frac{1}{2}\right)\pi = \frac{\pi}{2}$$

$$I_3 = \left(1-\frac{1}{4}\right)\frac{\pi}{2} = \frac{3}{4}\,\frac{\pi}{2} = \frac{3}{8}\pi$$

$$\Rightarrow I(\theta) = \frac{4}{\pi}\cdot\frac{3}{8}\pi - \frac{2}{\pi}\cdot\frac{\pi}{2} = \frac{3}{2} - 1 = \frac{1}{2}$$

$$D\hat{\theta} = \frac{1}{n\frac{1}{2}} = \frac{2}{n}$$

$$D X_{(\frac{n+1}{2})} = \frac{1}{4n\frac{1}{\pi^2}} = \frac{\pi^2}{4n} \approx \frac{2.47}{n}$$

$\Rightarrow D\hat{\theta} < D X_{(\frac{n+1}{2})}$

$\Rightarrow \hat{\theta}$ је боља оцена