$\boxed{153}$ $H_0(m_1 = m_2 = \dots = m_j)$ $H_1(m_i \neq m_j, i \neq j)$

$\boxed{j=2}$ $H_0(m_1 = m_2)$ $H_1(m_1 \neq m_2)$

$X \in \mathcal{N}(m_1, \sigma^2)$ $Y \in \mathcal{N}(m_2, \sigma^2)$

$n = n_1 + n_2$

n_1 - обим узорка за обележје X

n_2 - обим узорка за обележје Y

$$\Lambda = \frac{L(\hat{\hat{m}}, \hat{\hat{\sigma}}^2)}{L(\hat{m}, \hat{\sigma}^2)} \qquad \boxed{\hat{m}_1 = \bar{X}_{n_1}, \ \hat{m}_2 = \bar{Y}_{n_1}}$$

$$L(m_1, m_2, \sigma^2) = \left(\frac{1}{\sqrt{2\pi}}\right)^{n_1+n_2}\left(\frac{1}{\sigma^2}\right)^{\frac{n_1+n_2}{2}} e^{-\frac{1}{2\sigma^2}\left(\Sigma(x_k - m_1)^2 + \Sigma(y_k - m_2)^2\right)}$$

$$\ln L(m_1, m_2, \sigma^2) = -\frac{n_1+n_2}{2}\ln 2\pi - \frac{n_1+n_2}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\left(\Sigma(x_k - m_1)^2 + \Sigma(y_k - m_2)^2\right)$$

$$\ln L'_{\sigma^2}(m_1, m_2, \sigma^2) = -\frac{n_1+n_2}{2\sigma^2} + \frac{1}{2\sigma^4}\left(\Sigma(x_k - m_1)^2 + \Sigma(y_k - m_2)^2\right) = 0$$

$$\hat{\sigma}^2 = \frac{1}{n_1+n_2}\left(\Sigma(X_k - \hat{m}_1)^2 + \Sigma(Y_k - \hat{m}_2)^2\right)$$
$$= \frac{1}{n_1+n_2}\left(\Sigma(X_k - \bar{X}_{n_1})^2 + \Sigma(Y_k - \bar{Y}_{n_2})^2\right)$$

при H_0: $L(m, \sigma^2) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{1}{2\sigma^2}\left(\Sigma(x_k - m)^2 + \Sigma(y_k - m)^2\right)}$

$$\ln L(m, \sigma^2) = -\frac{n}{2}\ln 2\pi - \frac{n}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\left(\Sigma(x_k - m)^2 + \Sigma(y_k - m)^2\right)$$

$$\ln L'_m(m, \sigma^2) = -\frac{1}{2\sigma^2}\left(-2\Sigma(x_k - m) - 2\Sigma(y_k - m)^2\right) = 0$$

$$\Rightarrow \Sigma x_k - n_1 m + \Sigma y_k - n_2 m = 0$$

$$nm = \Sigma x_k + \Sigma y_k \Rightarrow \boxed{\hat{\hat{m}} = \frac{n_1\bar{X} + n_2\bar{Y}}{n}}$$

$$\hat{\hat{\sigma}} = \frac{1}{n}\left(\Sigma(X_k - \hat{\hat{m}})^2 + \Sigma(Y_k - \hat{\hat{m}})^2\right)$$

$$\Lambda = \frac{(2\pi\hat{\hat{\sigma}}^2)^{-\frac{n}{2}} \exp\left\{-\frac{1}{2\hat{\hat{\sigma}}^2}\left(\Sigma(x_k - \hat{\hat{m}})^2 + \Sigma(y_k - \hat{\hat{m}})^2\right)\right\}}{(2\pi\hat{\sigma}^2)^{-\frac{n}{2}} \exp\left\{-\frac{1}{2\hat{\sigma}^2}\left(\Sigma(x_k - \bar{x})^2 + \Sigma(y_k - \hat{m})^2\right)\right\}}$$

$$= \left(\frac{\hat{\sigma}^2}{\hat{\hat{\sigma}}^2}\right)^{\frac{n}{2}}$$

$$W = \{\Lambda \leq c\} = \left\{\frac{\frac{1}{n_1+n_2}\left(\Sigma(x_k - \bar{x})^2 + \Sigma(y_k - \bar{y})^2\right)}{\frac{1}{n_1+n_2}\left(\Sigma(x_k - \hat{\hat{m}})^2 + \Sigma(y_k - \hat{\hat{m}})^2\right)} \leq c_1\right\}$$

$$= \left\{\frac{\Sigma(x_k - \bar{x})^2 + \Sigma(y_k - \bar{y})^2}{\Sigma(x_k \underbrace{- \frac{n_1\bar{x} + n_2\bar{y}}{n}}_{-\bar{x}+\bar{x}})^2 + \Sigma(y_k \underbrace{- \frac{n_1\bar{x} + n_2\bar{y}}{n}}_{-\bar{y}+\bar{y}})^2} \leq c_1\right\}$$

$$= \left\{\frac{\Sigma(x_k - \bar{x})^2 + \frac{n_1 n_2^2}{n^2}(\bar{x} - \bar{y})^2 + \Sigma(y_k - \bar{y})^2 + \frac{n_1^2 n_2}{n^2}(\bar{y} - \bar{x})^2}{\Sigma(x_k - \bar{x})^2 + \Sigma(y_k - \bar{y})^2} \geq \frac{1}{c_1}\right\}$$

$$= \left\{1 + \frac{\frac{n_1 n_2}{n^2}(n_2 + n_1)(\bar{x} - \bar{y})^2}{\Sigma(x_k - \bar{x})^2 + \Sigma(y_k - \bar{y})^2} \geq c_2\right\}$$

$$= \left\{\frac{\sqrt{\frac{n_1 n_2}{n}}\,|\bar{x} - \bar{y}|}{\sqrt{\frac{n_1\bar{s}_x^2 + n_2\bar{s}_y^2}{n_1 - 1 + n_2 - 1}}} \geq k\right\}$$

при H_0: $\bar{X} - \bar{Y} \sim \mathcal{N}\left(0, \frac{1}{n_1} + \frac{1}{n_2}\right) \Rightarrow \sqrt{\frac{n_1 n_2}{n}}(\bar{X} - \bar{Y}) \in \mathcal{N}(0,1)$

$$\Rightarrow \frac{\sqrt{\frac{n_1 n_2}{n}}(\bar{X} - \bar{Y})}{\sqrt{\frac{n_1\bar{s}_x^2 + n_2\bar{s}_y^2}{n_1 + n_2 - 2}}} \in t_{n_1+n_2-2}$$

$$\Rightarrow k = F^{-1}_{t_{n_1+n_2-2}}(1 - \alpha)$$