Njutnov interpolacioni polinom sa konačnim razlikama

Ekvidistantna tablica: $x_{i+1} - x_i = h$, $i = 0, 1..n$.

$$
\begin{array}{cc|ccccc}
x_0 & \underline{y_0} & & & & & \\
 & & \underline{\Delta f_0} & & & & \\
x_1 & y_1 & & \underline{\Delta^2 f_0} & & & \\
 & & \Delta f_1 & & \underline{\Delta^3 f_0} & & \\
x_2 & y_2 & & \Delta^2 f_1 & & & \\
\vdots & \vdots & \vdots & \vdots & & & \\
 & & & & & \dots & \underline{\underline{\underline{\Delta^n f_0}}} \\
x_{n-2} & y_{n-2} & & & & & \\
 & & \Delta f_{n-2} & & \underline{\underline{\Delta^3 f_{n-3}}} & & \\
x_{n-1} & y_{n-1} & & \underline{\underline{\Delta^2 f_{n-2}}} & & & \\
 & & \underline{\underline{\Delta f_{n-1}}} & & & & \\
x_n & \underline{\underline{y_n}} & & & & &
\end{array} .
$$

1. Broj tačaka: n+1
2. Konačne razlike reda $1, ...n$
3. Polinomi stepena n:
   - **Prvi Njutnov interpolacioni polinom**

     $$x \in (x_0, x_1) \text{ ili } x < x_0, \quad q = \frac{x - x_0}{h}$$

     $$P_n^I(x) = f_0 + \Delta f_0 q + \frac{1}{2!}\Delta^2 f_0 q(q-1) + ... + \frac{1}{n!}\Delta^n f_0 q(q-1)...(q-n+1).$$

   - **Drugi Njutnov interpolacioni polinom**

     $$x \in (x_{n-1}, x_n) \text{ ili } x > x_n, \quad q = \frac{x - x_n}{h}$$

     $$P_n^{II}(x) = f_n + \Delta f_{n-1} q + \frac{1}{2!}\Delta^2 f_{n-2} q(q+1) + ... + \frac{1}{n!}\Delta^n f_0 q(q+1)...(q+n-1).$$

MATLAB

$$
\begin{array}{cc|cccccc}
x_1 & \underline{y_1} & \underline{\Delta f_1} & \underline{\Delta^2 f_1} & \underline{\Delta^3 f_1} & \dots & \underline{\Delta^{n-1} f_1} \\
x_2 & y_2 & \Delta f_2 & \Delta^2 f_2 & \Delta^3 f_2 & & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \\
x_{n-2} & y_{n-2} & \Delta f_{n-2} & \Delta^2 f_{n-2} & 0 & & \\
x_{n-1} & y_{n-1} & \Delta f_{n-1} & 0 & 0 & \vdots & \\
x_n & y_n & 0 & 0 & 0 & \dots & 0 \\
\hline
\text{X'} & \text{Y'} & & & \text{matrica KR} & &
\end{array} .
$$

1. Broj tačaka: n(=length(X))
2. Konačne razlike reda 1,2,...n-1.
3. Matrica KR dimenzije $n \times (n-1)$ za konačne razlike reda 1,2,...n-1 (j-ta kolona sadrži konačne razlike reda j kojih ima n-j, ostatak kolone je dopunjen nulama)
4. Polinom je stepena n-1, tj vektor duzine n

- **Inverzna interpolacija II Njutnov polinomom trećeg stepena**

$$x \in (x_{n-1}, x_n) \text{ ili } x > x_n, \quad q = \frac{x - x_n}{h}$$
$$P_n^{II}(x) = f_n + \Delta f_{n-1} q + \frac{1}{2!}\Delta^2 f_{n-2} q(q+1) + \frac{1}{3!}\Delta^3 f_{n-3} q(q+1)(q+2) = y.$$
$$q = \frac{-1}{\Delta f_{n-1}}\left(-y + f_n + \frac{1}{2!}\Delta^2 f_{n-2} q(q+1) + \frac{1}{3!}\Delta^3 f_{n-3} q(q+1)(q+2)\right) = \varphi(q)$$

$$q_0 = 0, \quad q_{n+1} = \varphi(q_n)$$
$$x = X(n) + q(end)h$$