ZADACI:

1. $\int \frac{\sqrt[3]{x}+1}{x} dx$

2. $\int \cos(4x) dx$

3. $\int \frac{e^x}{1+e^{2x}} dx$

4. $\int \frac{\operatorname{arctg} x}{1+x^2} dx$

5. $\int \operatorname{tg} x dx = \int \frac{\sin x}{\cos x} dx$

6. $\int \frac{x dx}{x^2+5}$

7. $\int \frac{dx}{2x^2-9x+4}$

8. $\int \frac{x dx}{(1+x^2)^2}$

9. $\int \frac{e^x+1}{e^x+x} dx$

10. $\int \frac{x^5}{(1+x^3)^2} dx$

11. $\int \ln x dx$

12. $\int x \cos x dx$

13. $\int \operatorname{arctg} x dx$

14. $\int \frac{\ln x}{x^2} dx$

1.
$$\int \frac{\sqrt[3]{x}+1}{x}dx = \int x^{-\frac{2}{3}}dx + \int \frac{dx}{x} = 3x^{\frac{1}{3}} + \ln|x| + c$$

2.
$$\int \cos(4x)dx = \begin{cases} 4x = t \\ 4dx = dt \\ dx = \frac{dt}{4} \end{cases} = \int \cos t \frac{dt}{4} = \frac{1}{4}\sin(4x) + c$$

3.
$$\int \frac{e^x}{1+e^{2x}}dx = \int \frac{e^x}{1+(e^x)^2}dx = \begin{cases} e^x = t \\ e^x dx = dt \end{cases} = \int \frac{dt}{1+t^2} = \operatorname{arctg} t + c = \operatorname{arctg}(e^x) + c$$

4.
$$\int \frac{\operatorname{arctg} x}{1+x^2}dx = \begin{cases} \operatorname{arctg} x = t \\ \frac{1}{1+x^2}dx = dt \end{cases} = \int t dt = \frac{1}{2}t^2 + c = \frac{1}{2}(\operatorname{arctg} x)^2 + c$$

5.
$$\int \operatorname{tg} x dx = \int \frac{\sin x}{\cos x}dx = \begin{cases} \cos x = t \\ -\sin x dx = dt \\ \sin x dx = -dt \end{cases} = \int \frac{-dt}{t} = -\ln|\cos x| + c$$

6.
$$\int \frac{xdx}{x^2+5} = \begin{cases} x^2+5 = t \\ 2xdx = dt \\ xdx = \frac{dt}{2} \end{cases} = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|x^2+5| + c$$

7.
$$\int \frac{dx}{2x^2-9x+4} = \begin{cases} \text{Nule polinoma su } \frac{1}{2} \text{ i 4 pa je:} \end{cases} = \int \frac{dx}{2(x-\frac{1}{2})(x-4)} = \int \frac{dx}{(2x-1)(x-4)}$$

Tražimo A,B tako da $\frac{1}{(2x-1)(x-4)} = \frac{A}{2x-1} + \frac{B}{x-4}$.

$\frac{1}{(2x-1)(x-4)} = \frac{A}{2x-1} + \frac{B}{x-4} / \cdot (2x-1)(x-4)$

$1 = A(x-4) + B(2x-1)$

$1 = x(A+2B) - 4A - B$

uz x^1: $0 = A + 2B$

uz x^0: $1 = -4A - B$

Rešavanjem sistema dobija se $B = \frac{1}{7}$, $A = -\frac{2}{7}$.

Vratimo se na integral:

$$= \int (\frac{-\frac{2}{7}}{2x-1} + \frac{\frac{1}{7}}{x-4})dx = -\frac{2}{7}\int \frac{dx}{2x-1} + \frac{1}{7}\int \frac{dx}{x-4} = \begin{cases} 2x-1=t; x-4=u \\ 2dx = dt; dx = du \\ dx = \frac{dt}{2} \end{cases} = -\frac{1}{7}\int \frac{dt}{t} + \frac{1}{7}\int \frac{du}{u} =$$

$$= \frac{1}{7}\ln|2x-1| + \frac{1}{7}\ln|x-4| + C$$

8.

$$\int \frac{xdx}{(1+x^2)^2} = \begin{cases} 1+x^2 = t \\ 2xdx = dt \\ xdx = \frac{dt}{2} \end{cases} = \int \frac{\frac{dt}{2}}{t^2} = \frac{1}{2}\int t^{-2}dt = \frac{1}{2}\cdot\frac{-1}{t} + c = \frac{1}{2}\cdot\frac{-1}{1+x^2} + c$$

9.

$$\int \frac{e^x+1}{e^x+x}dx = \begin{cases} e^x + x = t \\ (e^x+1)dx = dt \end{cases} = \int \frac{dt}{t} = \ln|e^x + x| + c$$

10.

$$\int \frac{x^5}{(1+x^3)^2}dx = \begin{cases} 1+x^3 = t \end{cases} = (\text{pre smene primetimo}) = \int \frac{x^3\cdot x^2dx}{(1+x^3)^2} = \begin{cases} x^3 = t-1 \\ 3x^2dx = dt \\ x^2dx = \frac{dt}{3} \end{cases} = \int \frac{(t-1)\frac{dt}{3}}{t^2}$$

$$= \frac{1}{3}(\int \frac{1}{t}dt - \int t^{-2}dt) = \frac{1}{3}(\ln|t| - (-\frac{1}{t})) + c = \frac{1}{3}(\ln|1+x^3| + \frac{1}{1+x^3}) + c$$

11.

$$\int \ln x dx = \int \ln x\cdot 1\cdot dx = \begin{cases} U = \ln x, V' = 1 \\ U' = \frac{1}{x}, V = x \end{cases} = x\ln x - \int x\frac{1}{x}dx = x\ln x - x + c$$

12.

$$\int x\cos x dx = \begin{cases} U = x, V' = \cos x \\ U' = 1, V = \sin x \end{cases} = x\sin x - \int 1\cdot\sin x dx = x\sin x + \cos x + c$$

13.

$$\int \operatorname{arctg} x dx = \int \operatorname{arctg} x\cdot 1 dx = \begin{cases} U = \operatorname{arctg} x, V' = 1 \\ U' = \frac{1}{1+x^2}, V = x \end{cases} = x\operatorname{arctg} x - \int \frac{x}{1+x^2}dx = \begin{cases} 1+x^2 = t \\ 2xdx = dt \\ xdx = \frac{dt}{2} \end{cases} =$$

$$= x\operatorname{arctg} x - \int \frac{\frac{dt}{2}}{t} = x\operatorname{arctg} x - \frac{1}{2}\ln|1+x^2| + c$$

14.

$$\int \frac{\ln x}{x^2}dx = \begin{cases} U = \ln x, V' = \frac{1}{x^2} \\ U' = \frac{1}{x}, V = -\frac{1}{x} \end{cases} = -\frac{1}{x}\ln x - \int \frac{1}{x}\cdot(-\frac{1}{x})dx = -\frac{1}{x}\ln x + \int x^{-2}dx = -\frac{1}{x}\ln x - \frac{1}{x} + c$$